Scheme Homework #2

1. pi is an already defined constant that WeScheme knows about -- try the expression: (/ pi 2). Some mathematicians think that pi is over-rated and would rather like to be working with a different constant called "tau" which is equal to 2 times pi. Their argument is that many mathematical and physical equations use "2*pi", so let's make that a named constant for that as well and use it. Write the Scheme code for defining tau. If you're interested in this controversy, see "Let's use Tau – it's easier than pi".

2. Below, you'll see an object called a "continued fraction". Show the expression to evaluate the answer and the answer itself. BTW, does the answer look familiar? Is it what you think it is? If not, why not? (write your answers)

$$3 + \frac{1}{7 + \frac{1}{16}}$$

3. One "Astronomical Unit" is defined to be the average distance from the Earth to the Sun. It is approximately 93 million miles. If light travels at approximately 186,000 miles/second, about how many minutes will it take light from the Sun to reach the Earth. Show the expression for the answer and also the answer itself (and round to 3 significant digits yourself)?

4. Create the scheme function called Circ(r) which is given the value of the radius of a circle and returns its circumference. Use tau. Isn't it simpler? Example: (Circ 3) -> 18.84955592153876

5. Create and show the two functions F2C(fahr) and C2F(cels), which convert temperature from Fahrenheit to Celsius (F2C), and Celsius to Fahrenheit (C2F). Examples for checking:
   (F2C 32) -> 0
   (C2F -40) -> -40
   (F2C (C2F 23.5)) -> 23.5

6. You're given a point in the plane at position (x,y). Create the function *dist(x,y)* that computes the distance from that point to the

origin. Think about using the Pythagorean Theorem. Example for checking: (dist 3 4) -> 5

7. <u>Challenge problem</u>: Let's create a mathematical (not Scheme) notation: "A (mod B)" is the remainder when A is divided by B. Now let's create a function called fermat(x,y,z) which takes 3 arguments that calculates the following:

$$fermat(x, y, z) = x^y (mod\ \ z)$$

For instance, $fermat(3,4,5) = 3^4(mod\ \ 5) = 81(mod\ \ 5) = 1$ because the remainder of 81 divided by 5 is 1.

Now let's notice a pattern found by the great mathematician Pierre de Fermat back in 1640. Evaluate the 4 examples below, each of which is in the form: X^Y (mod Z).

There's nothing special about the choice of X, but there **<u>is</u>** something special about the choice of Z and of Y. Guess what's special about the relationship between Y and Z, and what's special about Z? Write a sentence or two about your guesses…
2a) evaluate 2^{12} (mod 13) = fermat(2,12,13) = ?
2b) evaluate 5^{22} (mod 23) = fermat(5,22,23) = ?
2c) evaluate 55^{6} (mod 7) = fermat(55,6,7) = ?
2d) evaluate 18^{42} (mod 43) = fermat(18,42,43) = ?

First: guess what's special about the relationship between Y and Z, and what's special about Z…Write down your guess as one part of your answer to this question.
Second: test your guess with other numbers
Third: Try another guess (and write it down) if the first one wasn't quite correct…